

# **LECTURE 1 OP-AMP**

**Introduction of Operation Amplifier (Op-Amp)**

**Analysis of ideal Op-Amp applications**

**Comparison of ideal and non-ideal Op-Amp**

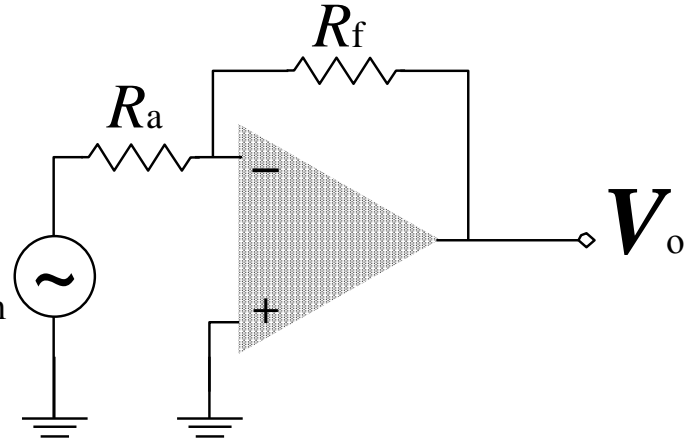
**Non-ideal Op-Amp consideration**



# INVERTING AMPLIFIER

(1) Kirchhoff node equation at  $V_+$  yields,  $V_+ = 0$

(2) Kirchhoff node equation at  $V_-$  yields,  $\frac{V_{in} - V_-}{R_a} + \frac{V_o - V_-}{R_f} = 0$



(3) Setting  $V_+ = V_-$  yields

$$\frac{V_o}{V_{in}} = -\frac{R_f}{R_a}$$

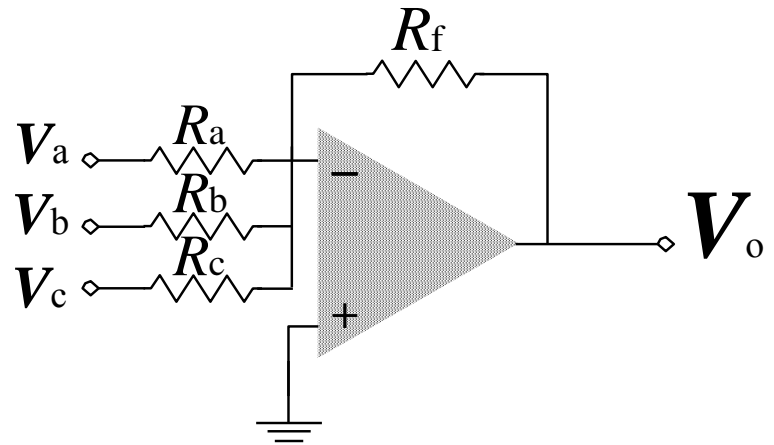
Notice: The **closed-loop gain**  $V_o/V_{in}$  is dependent upon the ratio of two resistors, and is independent of the open-loop gain. This is caused by the use of feedback output voltage to subtract from the input voltage.

# MULTIPLE INPUTS

(1) Kirchhoff node equation at  $V_+$  yields,  $V_+ = 0$

(2) Kirchhoff node equation at  $V_-$  yields,

$$\frac{V_- - V_o}{R_f} + \frac{V_- - V_a}{R_a} + \frac{V_- - V_b}{R_b} + \frac{V_- - V_c}{R_c} = 0$$



(3) Setting  $V_+ = V_-$  yields

$$V_o = -R_f \left( \frac{V_a}{R_a} + \frac{V_b}{R_b} + \frac{V_c}{R_c} \right) = -R_f \sum_{j=a}^c \frac{V_j}{R_j}$$

# INVERTING INTEGRATOR

Now replace resistors  $R_a$  and  $R_f$  by complex components  $Z_a$  and  $Z_f$ , respectively, therefore

Supposing 
$$V_o = \frac{-Z_f}{Z_a} V_{in}$$

(i) The feedback component is a capacitor  $C$ ,  
i.e.,

$$Z_f = \frac{1}{j\omega C}$$

(ii) The input component is a resistor  $R$ ,  $Z_a = R$

Therefore, the closed-loop gain ( $V_o/V_{in}$ ) become:

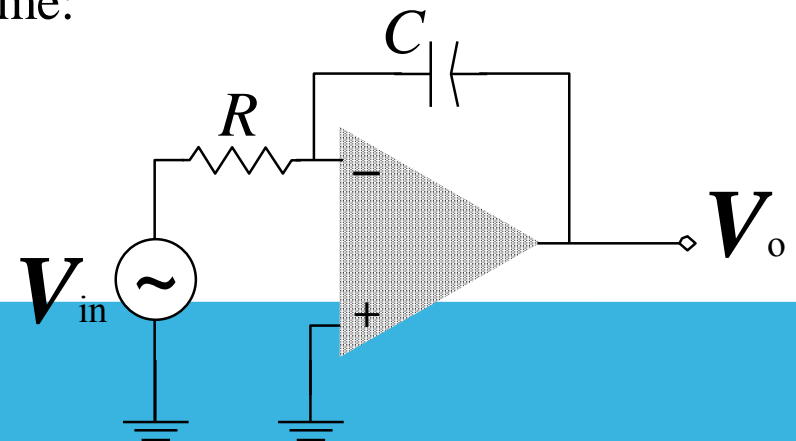
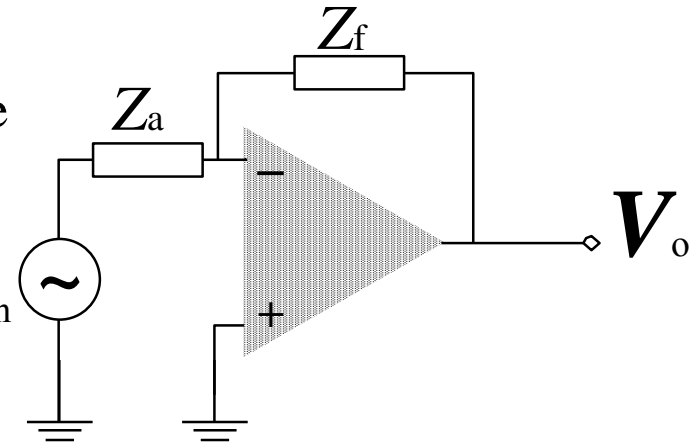
$$v_o(t) = \frac{-1}{RC} \int v_i(t) dt$$

where

$$v_i(t) = V_i e^{j\omega t}$$

What happens if  $Z_a = 1/j\omega C$  whereas,  $Z_f = R$ ?

*Inverting differentiator*



# OP-AMP INTEGRATOR

Example:

- (a) Determine the rate of change of the output voltage.

- (b) Draw the output waveform.

Solution:

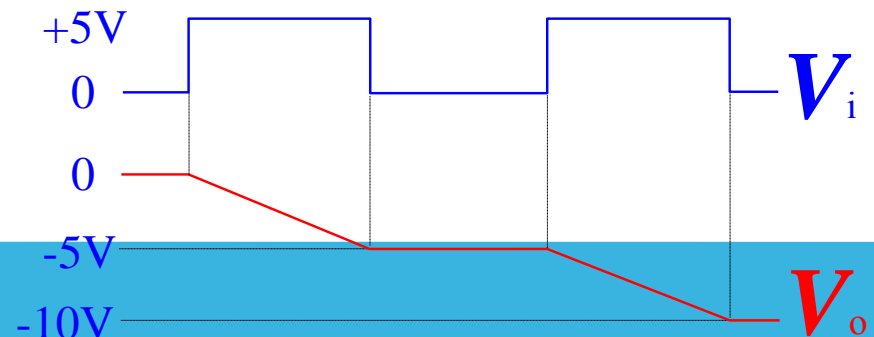
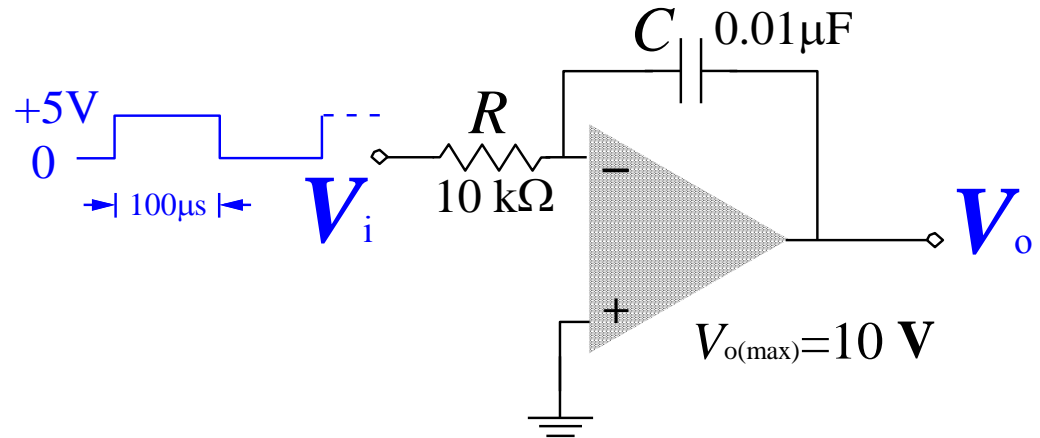
- (a) Rate of change of the output voltage

$$\frac{\Delta V_o}{\Delta t} = -\frac{V_i}{RC} = \frac{5 \text{ V}}{(10 \text{ k}\Omega)(0.01 \text{ }\mu\text{F})}$$

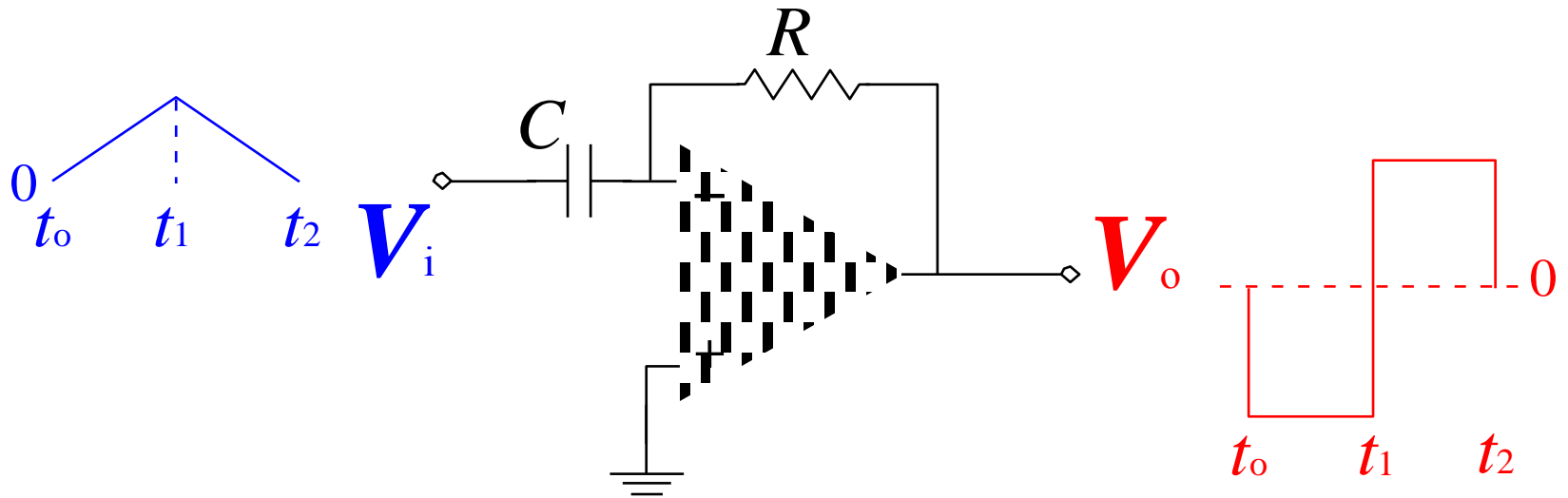
$$= -50 \text{ mV}/\mu\text{s}$$

- (b) In  $100 \text{ }\mu\text{s}$ , the voltage decrease

$$\Delta V_o = (-50 \text{ mV}/\mu\text{s})(100 \mu\text{s}) = -5 \text{ V}$$

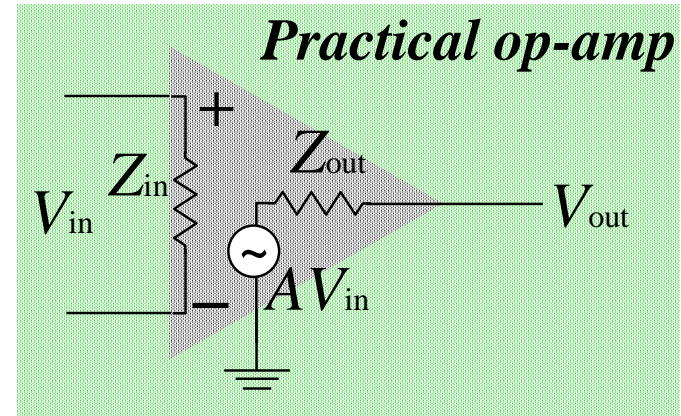
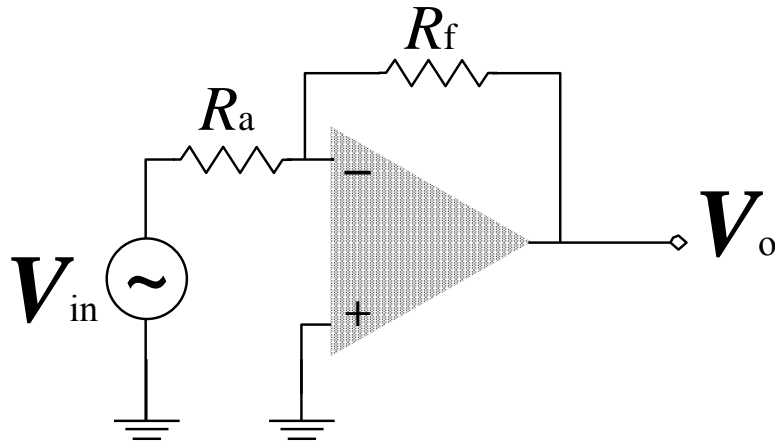


# OP-AMP DIFFERENTIATOR

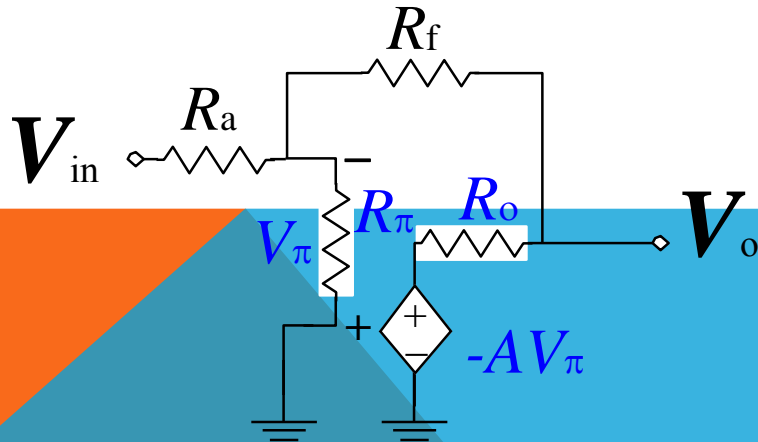


$$v_o = -\left(\frac{dV_i}{dt}\right)RC$$

# NON-IDEAL CASE (INVERTING AMPLIFIER)



⇓ Equivalent Circuit



3 categories are considering

- ☐ Close-Loop Voltage Gain
- ☐ Input impedance
- ☐ Output impedance

# CLOSE-LOOP GAIN

Applied KCL at V<sub>-</sub> terminal,

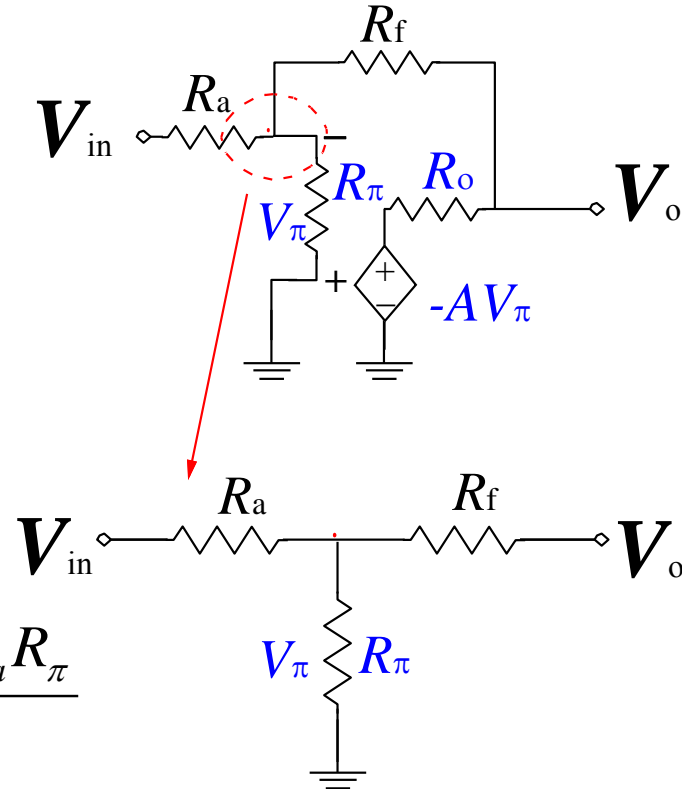
$$\frac{V_{in} - V_{\pi}}{R_a} + \frac{-V_{\pi}}{R_{\pi}} + \frac{V_o - V_{\pi}}{R_f} = 0$$

By using the open loop gain,

$$V_o = -AV_{\pi}$$

$$\Rightarrow \frac{V_{in}}{R_a} + \frac{V_o}{AR_a} + \frac{V_o}{AR_{\pi}} + \frac{V_o}{R_f} + \frac{V_o}{AR_f} = 0$$

$$\Rightarrow \frac{V_{in}}{R_a} = -V_o \frac{R_{\pi}R_f + R_aR_f + R_aR_{\pi} + AR_aR_{\pi}}{AR_aR_{\pi}R_f}$$



The Close-Loop Gain,  $A_v$

$$A_v = \frac{V_o}{V_{in}} = \frac{-AR_{\pi}R_f}{R_{\pi}R_f + R_aR_f + R_aR_{\pi} + AR_aR_{\pi}}$$



# CLOSE-LOOP GAIN

When the open loop gain is very large, the above equation become,

$$A_v \sim \frac{-R_f}{R_a}$$

Note : The close-loop gain now reduce to the same form as an ideal case

# INPUT IMPEDANCE

Input Impedance can be regarded as,

$$R_{in} = R_a + R_\pi // R'$$

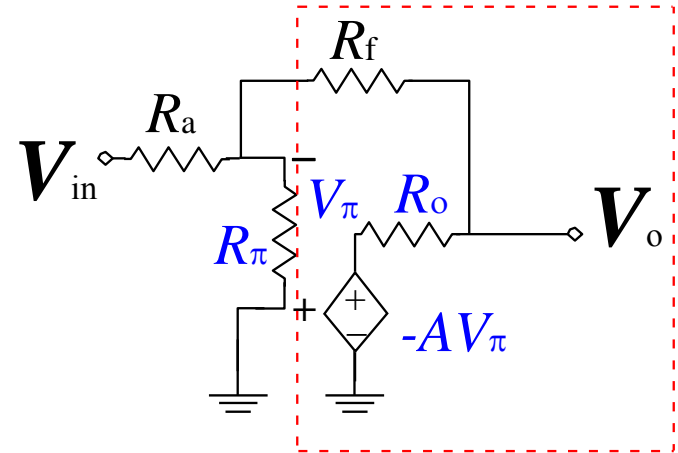
where  $R'$  is the equivalent impedance of the red box circuit, that is

$$R' = \frac{V_\pi}{i_f}$$

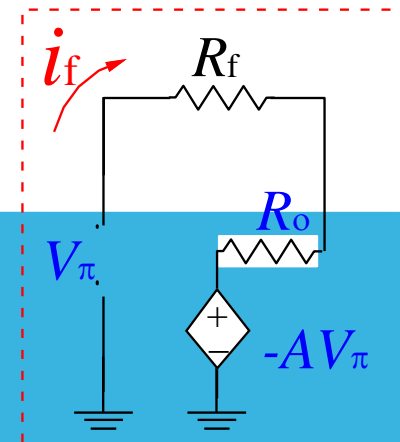
However, with the below circuit,

$$V_\pi - (-AV_\pi) = i_f (R_f + R_o)$$

$$\Rightarrow R' = \frac{V_\pi}{i_f} = \frac{R_f + R_o}{1 + A}$$



$R'$



# INPUT IMPEDANCE

Finally, we find the input impedance as,

$$R_{in} = R_a + \left[ \frac{1}{R_\pi} + \frac{1+A}{R_f + R_o} \right]^{-1} \Rightarrow R_{in} = R_a + \frac{R_\pi (R_f + R_o)}{R_f + R_o + (1+A)R_\pi}$$

Since,  $R_f + R_o \ll (1+A)R_\pi$ ,  $R_{in}$  become,

$$R_{in} \sim R_a + \frac{(R_f + R_o)}{(1+A)}$$

Again with  $R_f + R_o \ll (1+A)$

$$R_{in} \sim R_a$$

**Note:** The op-amp can provide an impedance isolated from input to output

# OUTPUT IMPEDANCE

Only source-free output impedance would be considered,  
i.e.  $V_i$  is assumed to be 0

Firstly, with figure (a),

$$V_\pi = \frac{R_a \parallel R_\pi}{R_f + R_a \parallel R_\pi} V_o \Rightarrow V_\pi = \frac{R_a R_\pi}{R_a R_f + R_a R_\pi + R_f R_\pi} V_o$$

By using KCL,  $i_o = i_1 + i_2$

$$i_o = \frac{V_o}{R_f + R_a \parallel R_\pi} + \frac{V_o - (-AV_\pi)}{R_o}$$

By substitute the equation from Fig. (a),

The output impedance,  $R_{out}$  is

$$\frac{V_o}{i_o} = \frac{R_o (R_a R_f + R_a R_\pi + R_f R_\pi)}{(1 + R_o)(R_a R_f + R_a R_\pi + R_f R_\pi) + (1 + A)R_a R_\pi}$$

$\therefore R_\pi$  and  $A$  comparably large,

$$R_{out} \sim \frac{R_o (R_a + R_f)}{AR_a}$$

